The study on the numerical computation of magnetic field coupled with fluid field in a magnetic fluid sensor

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Abstract — Advances of study in the field of ferrofluids, offer opportunities for many novel applications, such as various sensor applications. The distribution of magnetic field coupled with flow field has a direct influence on the sensor characters. In this paper, the dynamic equation of magnetic fluid is established and the coupling field equation is solved with finite element method, followed the programming calculation. Finally, the distribution of magnetic field coupled with flow field of a novel and intelligent magnetic fluid sensor is obtained. The result shows that the increase of the velocity of the permanent magnet has little effect on the magnetic field, but in contrast it adds to the flow velocity of the magnetic fluid in the axial gaps between the permanent magnet and the inner surface of the tube.

I. Introduction

Magnetic fluids (also known as ferrofluids or magnetic nanofluids) are stable suspensions of nanometric ferrimagnetic or ferromagnetic particles in suitable carrier fluids. It is a new functional nanomaterial which both has the properties of magnetic object and fluid. Since an external magnetic field can easily influence the location and properties of these fluids, they have been used in many scientific, industrial, and commercial applications such as magneto fluidic seals, lubricants, density separation, inkjet printers, refrigeration, diagnostics in medicine, clutches, tunable dampers, etc [1].

Advances of study in the field of ferrofluids offer opportunities for many novel applications, such as various sensor applications [2]-[4].

Based on second order levitation feature of magnet immersed in magnetic fluid, which makes the magnetic body avoid the mechanical contact with other bodies and move more freely and accurately, an accelerometer sensor [1] was proposed and the schematic diagram of the magnetic fluid sensor is shown in Fig. 1. The magnet represents an inertial mass and the sensor uses the displacement of the levitated magnet to measure acceleration.

However, there is no research concerning the distribution of magnetic field coupled with flow field when the permanent magnet moves at different speeds, which has a direct influence on the sensor characters.

In this paper, the dynamic equation of magnetic fluid was established and the coupling field equation was solved with finite element method, followed the programming calculation. Finally, the distribution of magnetic field coupled with flow field when the permanent magnet moves at different speeds was obtained and conclusions were drawn based on the analysis of the results.

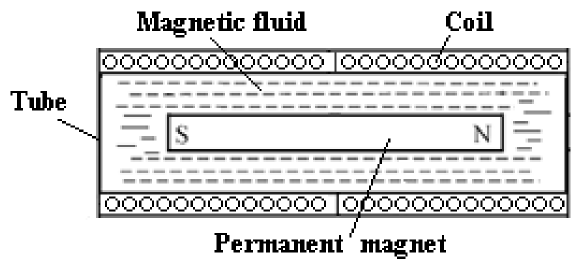

Fig. 1. Schematic diagram of the magnetic fluid sensor

II. Fundamental Principles

The behavior of the fluid is described by the equation of Navier-Stokes(N-S)， and the N-S equation of magnetic fluid was given by [5] which is

$$\rho \frac{d\boldsymbol{U}}{dt} = \boldsymbol{f}_g + \boldsymbol{f}_m + \boldsymbol{f}_\eta + \boldsymbol{f}_p \qquad (1)$$

where ρ is the density of the fluid, t the time, $\boldsymbol{U}$ the speed of the fluid, $\boldsymbol{f}_g$ the gravity, $\boldsymbol{f}_m$ the magnetic force, $\boldsymbol{f}_\eta$ the viscous force, and $\boldsymbol{f}_p$ is the pressure force. Among them, $\boldsymbol{f}_p$、 $\boldsymbol{f}_g$ and $\boldsymbol{f}_m$ belong to body force, and $\boldsymbol{f}_\eta$ belongs to surface force.

The N-S equation of magnetic fluid and that of ordinary fluid share the same gravity and the pressure force. While the differences lie in the magnetic force $\boldsymbol{f}_m$ and the viscous force $\boldsymbol{f}_\eta$ which are influenced by the magnetic field changes.

A. The Magnetic Force

The magnetic force per unit volume corresponding to a magnetic stress tensor $\boldsymbol{T}_m$ is [5]

$$\boldsymbol{f}_m = \nabla \bullet \boldsymbol{T}_m \qquad (2)$$

and $\boldsymbol{T}_m$ is defined as

$$\boldsymbol{T}_\mathrm{m} = -\{\mu_0 \int_0^H (\frac{\partial(vM)}{\partial v})_{H,T} dH + \frac{1}{2}\mu_0 H^2\}\boldsymbol{I} + \boldsymbol{BH} \qquad (3)$$

where μ_0 is the permeability of free space, $\boldsymbol{H}$ the magnetic field intensity, $\boldsymbol{B}$ the magnetic flux density, M the intensity of magnetization, $v=\rho^{-1}$ the specific volume, and $\boldsymbol{I}$ is the unit dyad.

The magnetic force can thus be written as

$$\boldsymbol{f}_m = \nabla\bullet\boldsymbol{T}_m = -\nabla\bullet[\mu_0\int_0^H (\frac{\partial(vM)}{\partial v})_{H,T}dH + \frac{1}{2}\mu_0 H^2]\boldsymbol{I} + \nabla\bullet(\boldsymbol{HB}) \tag{4}$$

It is shown in Liu's research [6] that when the permeability μ of magnetic fluid is irrelevant to the magnetic field intensity, the magnetic force can be simplified as Helmholtz force which is

$$\boldsymbol{f}_m = \nabla[\frac{H^2}{2}\rho(\frac{\partial\mu}{\partial\rho})_T] - \frac{H^2}{2}\nabla\mu \tag{5}$$

While the average magnetic moment of the magnetic particles is irrelevant to the specific volume v of magnetic fluid, the magnetic force can be simplified as Kelvin force which is

$$\boldsymbol{f}_m = \mu_0 M(\nabla H) \tag{6}$$

B. The Viscous Force

Mark I. Shliomis [7] argued that the influence of the second viscous force can not be ignored even for an incompressible liquid. Also, the equation of the viscous force of magnetic fluid was proposed as

$$\boldsymbol{f}_\eta = \eta_0\nabla^2\boldsymbol{U} + \boldsymbol{f}_{\Delta\eta} \tag{7}$$

where η_0 is the viscosity coefficient of magnetic fluid without external magnetic field, and the first part is the first viscous force and $\boldsymbol{f}_{\Delta\eta}$ is the second viscous force.

Based on this equation, a model of the second viscous force was raised by [8] which is

$$\boldsymbol{f}_{\Delta\eta} = \frac{3}{2}\eta_0\phi\frac{\xi L(\xi)}{[2+\xi L(\xi)]}\nabla^2\boldsymbol{U} \tag{8}$$

where $\boldsymbol{f}_{\Delta\eta}$ is the second viscous force and φ is the volume fraction of magnetic particles. ξ, the dimensionless field strength, can be calculated as

$$\xi = \frac{\mu_0 mH}{k\,T} \tag{9}$$

where m is the single particle magnetic moment, k the Boltzmann constant and T is the temperature.

Equation (8) is obtained from theoretical calculation. It is reasonable to adopt this theory when the volume fraction of magnetic particles is low. However, when the volume fraction of magnetic particles is relatively high and particles begin to form chains, a correction is required.

So the model of the viscous force of magnetic fluid can be written as

$$\boldsymbol{f}_\eta = \boldsymbol{f}_{\eta 1} + \boldsymbol{f}_{\eta 2} = \eta_0\left[1+\frac{3}{2}\gamma\phi\frac{\xi L(\xi)}{2+\xi L(\xi)}\right]\nabla^2\boldsymbol{U} \tag{10}$$

where γ is the correction coefficient related to φ .

III. Calculation and Conclusion

According to the above equations of the magnetic force and the viscous force, the N-S equation can be simplified. Because the average magnetic moment of the magnetic particles is irrelevant to the specific volume, the magnetic force can be simplified as Kelvin force, for the magnetic fluid adopted in this paper.

The N-S equation can be written as

$$\rho\frac{d\boldsymbol{U}}{dt} = \rho g + \mu_0 M(\nabla H) + \eta_0\left[1+\frac{3}{2}\eta_0\phi\frac{\xi L(\xi)}{2+\xi L(\xi)}\right]\nabla^2\boldsymbol{U} - \nabla p \tag{11}$$

The coupling field equation was solved with finite element method, followed the programming calculation. Finally, the distribution of magnetic field coupled with flow field when the permanent magnet moves at different speeds was obtained.

The result shows that the increase of the velocity of the permanent magnet has little effect on the magnetic field, but in contrast it adds to the flow velocity of the magnetic fluid in axial gaps between the permanent magnet and the inner surface of the tube.

And the flow velocity near the two axial ends of the permanent magnet is extremely high while the flow velocity near the two axial ends of the tube is significantly small. In axial gaps between the surface of the permanent magnet and the inner surface of the tube, the flow velocity near both the permanent magnet surface and the inner surface of the tube is small while the flow velocity in the middle of the gaps is relatively higher.

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